Intro Video: Section 2.2 part 2
Infinite limits

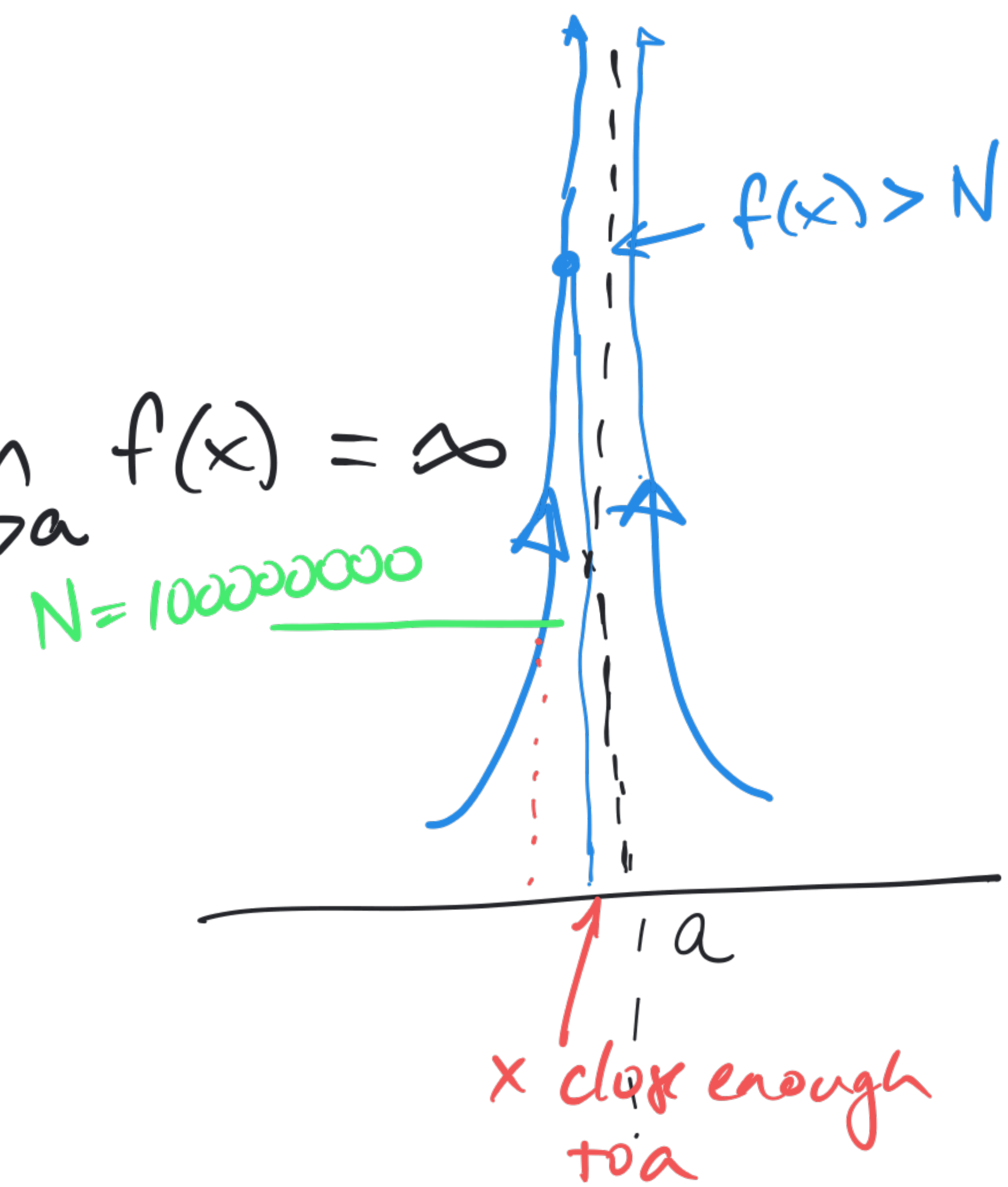
Math F251X: Calculus I

Infinite Limits

Suppose f is defined near a.

Intuitively, when we write $\lim_{x \to a} f(x) = \infty$

As $x \to a$, $f(x)$ gets arbitrarily large



VERTICAL ASYMPTOTE

If $\lim_{x \to a^+} f(x) = +\infty$ or $-\infty$

-or- $\lim_{x \to a^-} f(x) = +\infty$ or $-\infty$

then $x = a$ is a vertical

Example $f(x) = \frac{1}{x^2}$

What can we say about $\lim_{x \to 0} f(x)$?

As $x \to 0$ $\quad \frac{1}{x^2} \to 1$

$x = \frac{1}{100}$?

$x = \frac{-1}{\underbrace{100000}_{10^5}}$

$x^2 = \frac{1}{10000}$

$x^2 = \frac{+1}{\underbrace{10000000000}_{10^{10}}}$

$\frac{1}{x^2} = \frac{1}{\frac{1}{1000}} = 1000$

$\frac{1}{x^2} = \frac{1}{10^{10}}$
$= 10000000000$

$\frac{1}{x^2}$ looks like $\frac{1}{\text{really small}+}$

$$\boxed{\begin{array}{l} \lim_{x \to 0} \frac{1}{x^2} = \infty \\ x = 0 \text{ is a vertical asymp.} \end{array}}$$

= really big positive number!

Example $f(x) = \frac{5x}{x+2}$

Notice f is not defined at $x = -2$.

As $x \to -2^-$

$x = -2.001 = -2 - 0.001$

$5x \to -10 < 0$

$x + 2 < 0$

$x + 2 = -2 - 0.001 + 2 = -0.001 < 0$

So $\frac{5x}{x+2}$ looks like $\frac{-}{\text{really small negative}}$ which is a really big positive #!

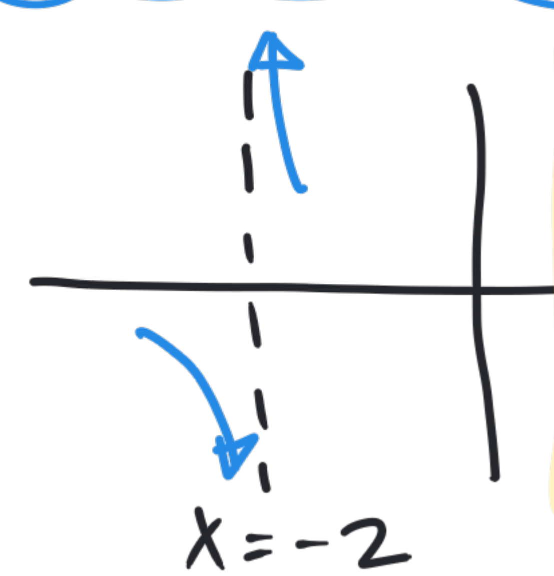
$\frac{5x}{x+2} = \frac{-10}{-0.001} = \frac{10}{\frac{1}{1000}} = 10000$

So $\lim_{x \to -2^-} \frac{5x}{x+2} = \infty$.

$\neq$

And as $x \to -2^+$, $\frac{5x \;\to -10}{x+2 \;\to 0^+}$

So $\lim_{x \to -2^+} \frac{5x}{x+2} = -\infty$



Notice $\lim_{x \to 2} \frac{5x}{x+2}$ DNE

Sometimes you need to be careful!

Use calculation to guess $\lim_{t \to 0} \frac{\sqrt{t^2+9} - 3}{t^2}$

"Looks like $\frac{0}{0}$"

We can't say anything yet about the behavior!

Let's look at evaluating

$f(t) \longrightarrow \frac{\sqrt{t^2+9} - 3}{t^2}$ for some values of t near 0.

t	$f(t)$
-0.001	0.166667
0.0001	0.166667
0.00001	0.166667
$1. \times 10^{-6}$	0.166533
$1. \times 10^{-7}$	0.177636
$1. \times 10^{-8}$	0.

$\longrightarrow 1/6$

???

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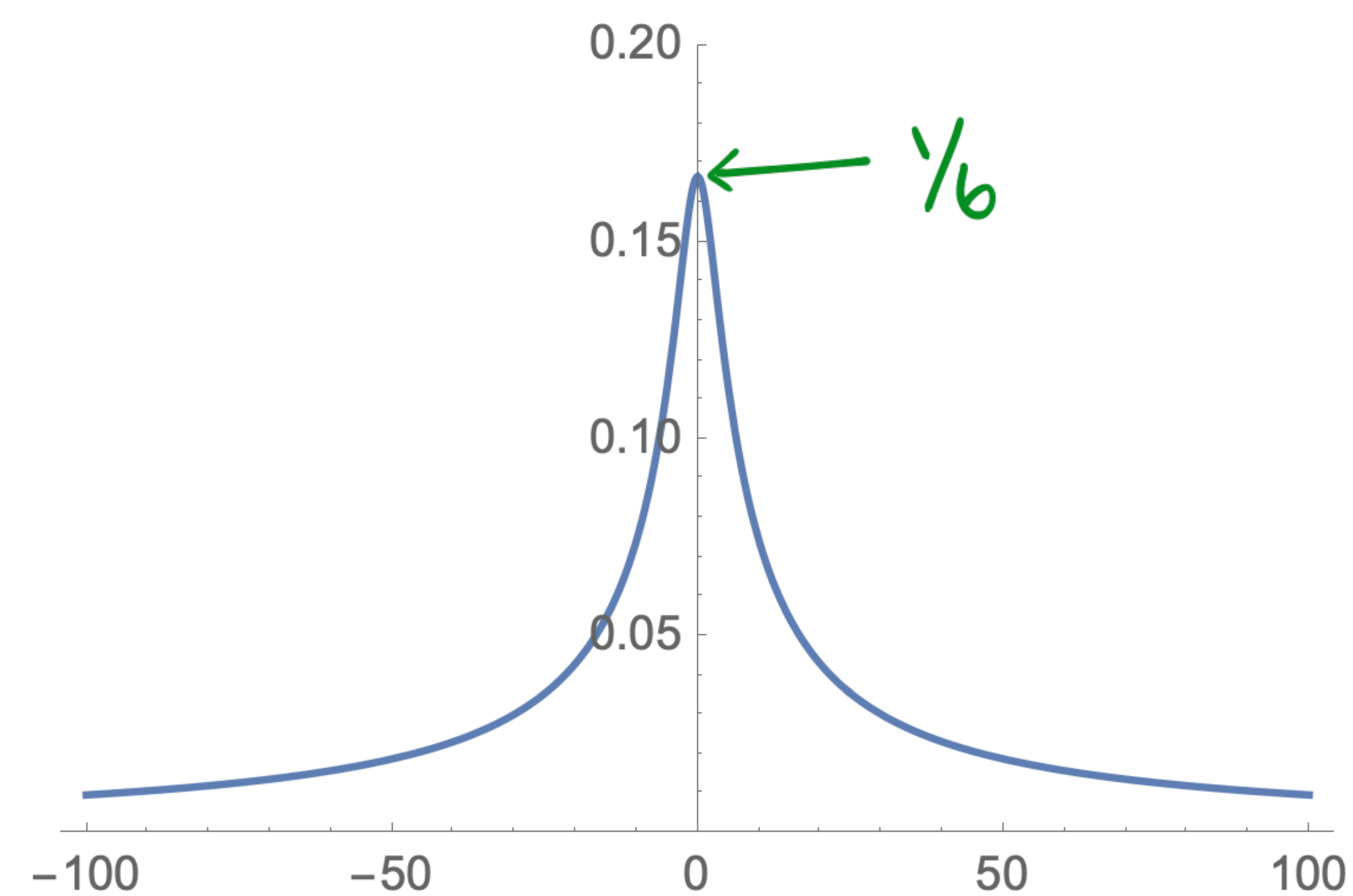


One more cautionary tale:

What can we say about $\lim_{\theta \to 0} f(\theta)$ where $f(\theta) = \sin\left(\frac{\pi}{\theta}\right)$

θ	$\frac{-1}{10}$	$\frac{-1}{100}$	$\frac{-1}{1000}$	0	$\frac{1}{1000}$	$\frac{1}{100}$	$\frac{1}{10}$
$f(\theta)$	0	0	0	DNE	0	0	0

$f\left(\frac{-1}{10}\right) = \sin\left(\frac{\pi}{-1/10}\right) = \sin(-10\pi) = 0$

$f\left(\frac{-1}{100}\right) = \sin\left(\frac{\pi}{-1/100}\right) = \sin(-100\pi) = 0$

$f\left(\frac{-1}{1000}\right) = \sin\left(\frac{\pi}{-1/1000}\right) = \sin(-1000\pi) = 0$

$f\left(\frac{1}{10}\right) = \sin\left(\frac{\pi}{1/10}\right) = \sin(10\pi) = 0$

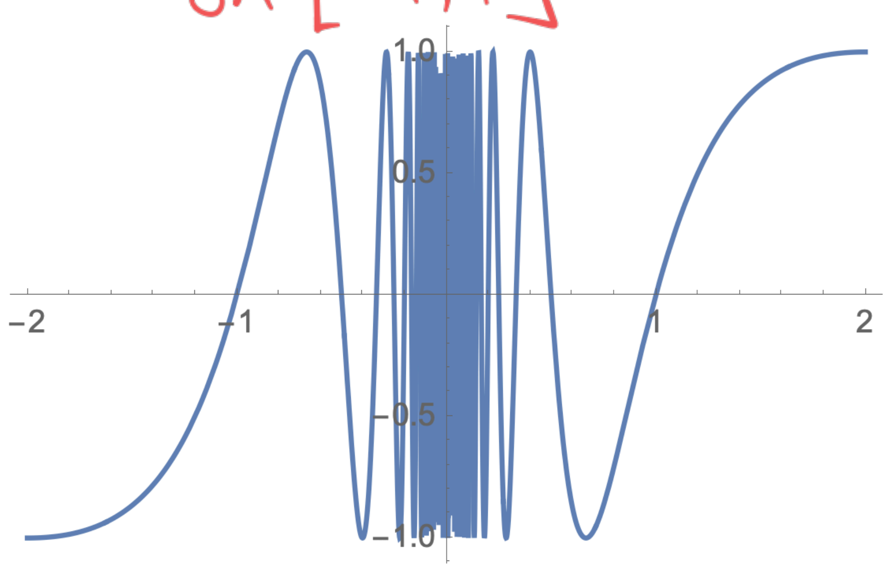
$f\left(\frac{1}{100}\right) = \sin(100\pi) = 0$

$f\left(\frac{1}{1000}\right) = \sin(1000\pi) = 0$

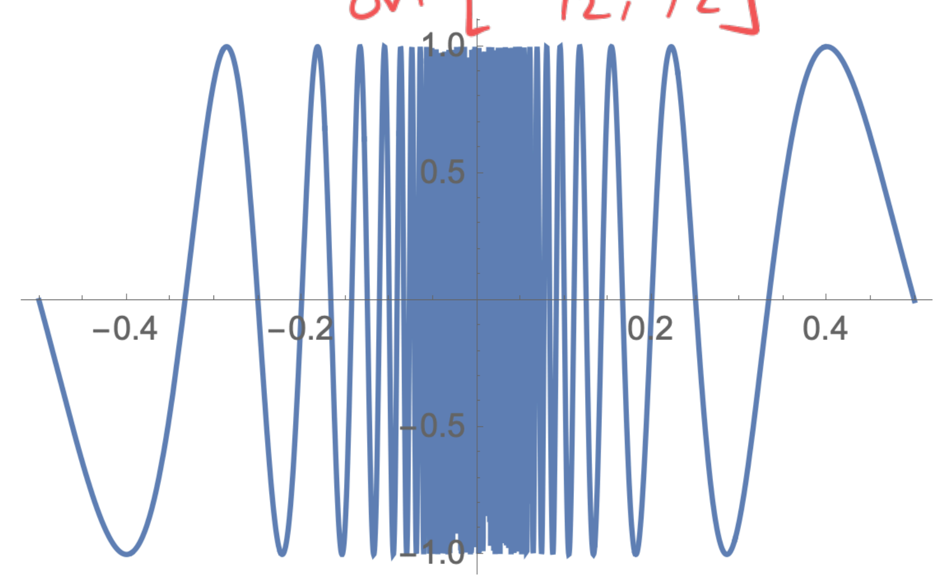
Is it true that $\lim_{\theta \to 0} \sin\left(\frac{\pi}{\theta}\right) = 0$? No!

$\boxed{\lim_{\theta \to 0} \sin\left(\frac{\pi}{\theta}\right) \text{ DNE}}$

on $[-1, 1]$



on $[-1/2, 1/2]$



on $[-1/10, 1/10]$